Impedance of a bootstrap coaxial trap

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Abstract

This article presents the derivation for an expression for the impedance of a bootstrap connected coax trap based on a transmission line model of the coax interior and model of the exterior inductor.

Figure 1: Bootstrap coax trap

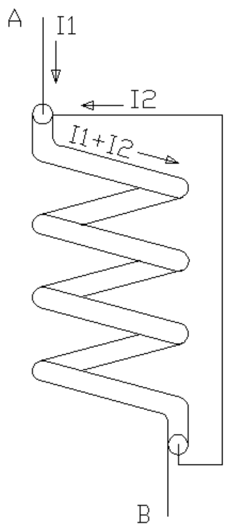


Fig 1 shows the configuration of a bootstrap connected coax trap, sometimes called the Hi-Z connection.

If it is assumed that skin effect is fully effective, a simple equivalent circuit of this structure comprises two elements:

- an inductor formed by the outer surface of the coil of coax; and
- the coax transmission line.

Inductor

The inductor can be modelled as having some inductance, and some equivalent series loss resistance. At the frequencies of interest, this model is inadequate in that it doesn't account for stray capacitance. A first approximation of stray capacitance is a small capacitance in parallel with the series combination of L and R.

Transmission Line

A transmission line of characteristic impedance Z_o can be represented by a two port network, with voltage V_1 and current I_1 into port 1, and voltage V_2 and current I_2 out of port 2.

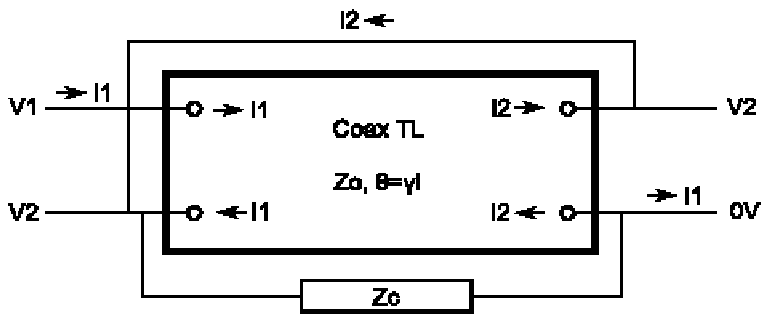
The differential mode of the transmission line can be described by the following pair of equations.

$$V_1 = V_2 \cdot cosh(\theta) + i_2 \cdot Z_o \cdot sinh(\theta) \tag{1}$$

$$I_1 = \frac{V_2}{Z_o} \cdot sinh(\theta) + i_2 \cdot cosh(\theta) \tag{2}$$

Bootstrap trap

Figure 2: Equivalent circuit of bootstrap trap



The inductor (Zc) and transmission line bootstrap connected can be represented by the equivalent circuit in Fig 2 where ϑ is the product of the complex propagation constant γ and length l, and Zc is the impedance of the choke formed by the outer surface of the outer conductor of the coax forming the trap as discussed above.

The following three equations can be written to describe the system.

$$V_2 = (I_1 + I_2) \cdot Z_c \tag{3}$$

$$V_1 - V_2 = V_2 \cdot cosh(\theta) + i_2 \cdot Z_o \cdot sinh(\theta) \tag{4}$$

$$I_1 = \frac{V_2}{Z_o} \cdot sinh(\theta) + i_2 \cdot cosh(\theta) \tag{5}$$

Note that V1 and V2 are with respect to the point 0V. The point V1 is equivalent to A in Fig 1 and 0V is equivalent to B.

Solving this set of equations for $Z_{trap} = V_1/I_1$ gives the following expression.

$$Z_{trap} = Z_o \cdot \frac{cosh\,(\theta) + 1}{cosh\,(\theta)} \cdot \frac{2 \cdot Z_c + Z_o \cdot tanh\left(\frac{\theta}{2}\right)}{Z_o + Z \cdot tanh(\theta)} \tag{6}$$

